IV. CONSTITUTIVE RELATIONS FOR IRON

4.1 Approximations for Two-Phase Flow

Three approximations were made in order to simplify the application of the principles of Section III to the $\alpha \rightarrow \epsilon$ transition in iron. First, the equilibrium volume change at constant p was assumed constant in the coexistence region. This approximation is supported by compressibility measurements at room temperature which result in the following values (10,28):

$$\beta_2 \text{ (at 192 Kb)} = 4.94 \times 10^{-1} \text{(Mb)}^{-1}$$

$$\beta_1 \text{ (at 132 Kb)} = 5.1 \times 10^{-1} \text{(Mb)}^{-1}$$

where β_i is the compressibility of the i-th phase.
Then

$$v_2(p,T) - v_1(p,T) = -.004 \text{ cc/g at every p and T.} \qquad (4.1)$$

Then from Eqs. (3.6), (3.26) and (4.1) we get:

$$dv_1 = dv - (v_2-v_1)d\alpha \qquad (4.2)$$

$$d\alpha = (\alpha^{eq}-\alpha)dt/\tau \qquad (4.3)$$

Then v_1 can be calculated from the above equations provided v and α^{eq} are known. In the computation process v is given by the continuity equation and α^{eq} by Eq. (3.6).

$$\alpha^{eq} = ((v-v_1)/(v_2-v_1))^{eq} = (v-v_1^{eq}(T))/(v_2-v_1)^{eq} \qquad (4.4)$$

Since $(v_2-v_1)^{eq}$ is constant, the temperature dependence of α^{eq} is in $v_1^{eq}(T)$. Our second approximation is to replace $v_1(T)$ by the value $v_1(T_o)$ at room temperature (T_o). Since the temperature is only slightly greater than room temperature, the error involved in this approximation is negligible except in the immediate vicinity of the transition pressure, 130 Kb.

Since v_1 can be determined from Eqs. (4.2) and (4.3) we can use the form $p = p(v_1,T)$ to calculate pressure if we know the temperature. The third approximation we made is the temperature calculation.

If the phase transition is made at equilibrium, pressure and temperature in the coexistence region must satisfy the relation:

$$dp/dT = \text{constant at fixed } p \text{ or } T.$$

Over a wide range of temperatures the coefficient is practically constant (10). But if we assume that the coefficient dp/dT is constant even in the non-equilibrium coexistence region, we shall see presently that the energy conservation law alone is then sufficient to determine the temperature change. Since E is given by Eq. (3.7) the internal energy is a function of p, T and α. On replacing α by Eq. (3.6), it is a function of p, T and v. But if we assume that dp/dT is constant, p and T are no longer independent and E is a function of T and v only. The